Exercise 12

In Exercises 9 to 12, describe all unit vectors orthogonal to both of the given vectors.

$$2\mathbf{i} - 4\mathbf{j} + 3\mathbf{k},\ -4\mathbf{i} + 8\mathbf{j} - 6\mathbf{k}$$

Solution

Each of the vectors can be written as

$$2\hat{\mathbf{x}} - 4\hat{\mathbf{y}} + 3\hat{\mathbf{z}} = (2, -4, 3)$$
$$-4\hat{\mathbf{x}} + 8\hat{\mathbf{y}} - 6\hat{\mathbf{z}} = (-4, 8, -6).$$

Notice that the second one is a constant multiple of the first: $-2(2, -4, 3) = (-4, 8, -6)$. Every unit vector lying in a plane with $(2, -4, 3)$ as its normal vector is orthogonal to both of the given vectors. The equation of the plane is given by

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$$

$$(2, -4, 3) \cdot (x - x_0, y - y_0, z - z_0) = 0$$
$$2(x - x_0) - 4(y - y_0) + 3(z - z_0) = 0$$
$$2x - 4y + 3z = C,$$

where C is a constant.